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Designing of Skip-Lot Sampling Plan of Type (Sksp-3) For Life Tests Based On Percentiles of Exponentiated Rayleigh Distribution

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Abstract- Skip-lot sampling plans aids in reducing the inspection cost. SkSP-3 plan is developed based on the principles of CSP-2.In this study, Skip lot sampling plan of type SkSP-3 with Single Sampling plan as reference plan is developed for the life testing based on the percentiles of Exponentiated Rayleigh Distribution (ERD). Comparative study on SkSP-3 with SSP as reference plan is also made. The operating characteristic values of the plan are tabulated and the curve is drawn. Numerical illustration is provided to validate the efficiency of the developed plan.

Keywords: Exponentiated Rayleigh Distribution, Percentiles, Life tests, Single Sampling Plan, SkSP -3.

1. INTRODUCTION

The acceptance sampling plans procedure involves accepting or rejecting a submitted lot based on the results of inspection of sample inspected taken from the lot. Acceptance sampling plan requires minimum sample size to for testing. If the characteristic of the quality is concerning the life time of the product, testing is termed as life testing. Acceptance sampling plan based on life testing generally applied to decide whether to accept or reject the products where the life span of it is a vital factor. Usually, when the results of the life testing shows that the mean life of the tested product exceeds pre-determined mean, the lot is accepted or else rejected. Life testing acceptance sampling plans are primarily assumed life time distribution, truncated scheme and testing conditions. The drawback in the life testing plans based on the assumed life distribution and testing condition is the determination of samples to be inspected. Truncated life test used in determination of smallest sample size, thus reducing the testing time and cost. This method involves the termination of life testing at a pre-determined time on the mean life of the tested products. The number of failures is observed, if it is above the specified acceptance number c, the lot is rejected otherwise accepted. Life testing plans based on truncated time provide better protection to the consumer as the conclusion to accept the lot is made only if specified mean life can be reached with a pre-determined high probability.

The life testing plans based on time truncated assumed under non-normal conditions are studied by many researchers for instance, Epstein (1954), , Goode and Kao (1961),Gupta and Groll (1961), Kantam *et al.* (2001, 2006), Baklizi (2003), Balakrishnan *et al*. (2007), Aslam and Shahbaz (2007), Aslam and Kantam (2008), Rao *etal.*(2008, 2009a, 2009b), Lio, Tsai and Wu (2010), Sriramachandran and Palanivel (2014) and Priyah and Sudamani (2015) . Most of the researchers consider life testing based on the mean. The drawback of life testing plans based on mean is that it may not fulfill engineering requirements on pre-determined strength or break stress. When applying plans based on mean on inspecting it may result in accepting bad quality of the products. This results in increased decorating of the products and will lead to the reduction of the life time of the products. Life testing plans based on median is more effective than the testing based on the life testing plans based on mean. So in this study, life testing plans based on 50[th] percentile (median) is considered.

Many characteristics of Exponentiated Rayleigh distribution is similar to that of gamma, Weibull and exponeniated exponential distribution. The distribution and density function of ERD are in close forms. As a result it is easily applied to the truncated plans. The cumulative distribution function of ERD is given by,

$$F(t;\tau,\theta) = \left[1 - e^{-1/2(t/\tau)^2}\right]^{\theta}, t > 0 \qquad 1/\tau>0,\ \theta>0 \qquad (1)$$

Where τ and θ are the scale and shape parameter respectively. The probability density function of ERD can be written as,

$$f(t;\tau,\theta)=\theta\left[1-e^{-1/2(t/\tau)^2}\right]^{\theta-1}\left[\frac{t}{\tau^2}e^{-1/2(t/\tau)^2}\right]. \quad (2)$$

Pradeepa Veerakumari and Ponneeswari (2016) proposed SSP for life testing based on the percentiles of ERD. Later, Pradeepa Veerakumari and Ponneeswari (2017) developed DSP for life testing based on the percentiles of ERD. The major objective of this study is to minimizing the sample size at specified quality levels. To reduce the usage of large samples it is always preferred to the use acceptance sampling plans, which safeguards consumer as well as producer from risk. In order to meet the requirements of less sample size with protection of consumer and producer, SkSP-3 with SSP as reference plan for life tests based on percentiles of ERD as reference plan is developed.

2. SKIP LOT SAMPLING PLAN

Dodge (1955) proposed skip-lot sampling plans based on the principle of continuous sampling plan of type CSP-1 for a series of lots or consignments of material .This plan is termed as SkSP-1 plan and it is applicable when there is a bulk material or products manufactured in successive lots. Later Perry (1973) introduced the concept of inspecting each lot according to the reference plan. Soundararajan and Vijayaraghavan (1987) developed a new sampling system SkSP-2 based on the principles of continuous sampling plan of type-2 of Dodge & Terry (1952) for the inspection of bulk products when there is continuous flow. SkSP-3 uses reference plan. Vijayaraghavan (2000) developed SkSP-3 with zero acceptance number SSP as reference plan using Markov chain. The modus operandi of the SkSP-3 plan is given below:

1. At the outset, starts with normally inspecting lot using the reference plan. At this level, the products are normally inspected according to the order of production or in the order submitted for inspection.
2. If i consecutive lots are accepted during normal inspection discontinue inspecting every lot and switch to skipping inspection.
3. Inspect only a fraction f of the submitted lots during skipping inspection level, until a lot is rejected.
4. If a lot is rejected while skipping inspection, then inspect next k lots produced or submitted.
5. Switch to the normal inspection, if a lot is rejected while inspecting k lots.
6. If all the k lots are accepted, continue skipping inspection.
7. Replace the non-conforming units in the rejected lot with conforming one.

SkSP-3 is characterized by three skipping parameters namely f,i & k, the value of f lies between 0 t0 1, i &k are positive integers. It involves three phases of inspection namely screening phase, limited sampling phase and unlimited sampling phase. The flow chart representing the modus operandi of the SkSP-3 is given below:

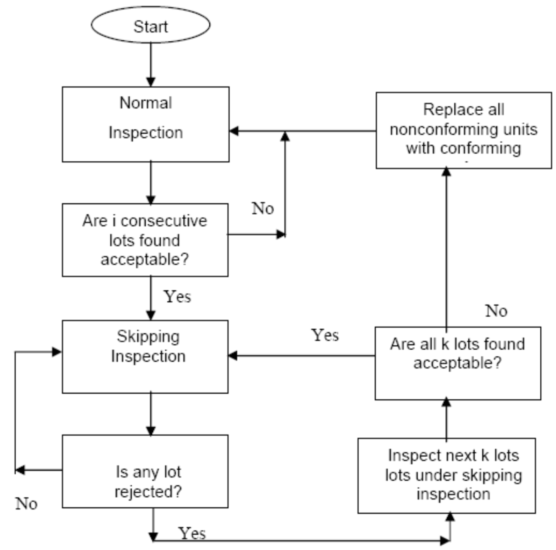

Figure1. Flow chart for operating procedure of SkSP-3 plan

3. Designing of Skip lot sampling plan of type SkSP-3 with SSP for life test based on the percentile of ERD as reference plan

Skip lot sampling plan of type 3(SkSP-3) has the provision to skip few lots when the quality standards of the submitted products are good and thus reduces the inspection cost. This plan makes use of basic attribute lot-by-lot acceptance sampling plans to inspect the individual lots, which is designated as reference plan.

SkSP-3 with SSP as reference plan for life tests based on percentiles of ERD is executed according to the procedure illustrated in 1.1. Incorporating the modus operandi of SSP for life tests based on percentiles of ERD to inspecting each lot. SSP for life tests based on the percentiles of ERD is characterized by sample size *n*, acceptance number *c* and failure probability *p* where $p=F(t,\delta_0)$.

The modus operandi of SkSP-3 with SSP as reference plan based on percentiles of ERD are as follows:

Step 1: A random sample of size n is drawn and put Draw a random sample of size and placed on test for time t_0.

Step 2: The number of defectives d are counted and comparison is made with the acceptance number c.

i. If $d > c$, then reject the lot.

ii. If $d \leq c$, then accept the lot.

Step 3: If $d > c$, is obtained before the specified time t_0, terminate the test and reject the lot.

3.1 Formation of the Sampling Plan

The crucial factor in designing a sampling plan is selection of plan parameters satisfying the consumer and producer's requirement. Two point OC curve approach is applied to satisfy the requirements ARL (p_0) and LRL (p_1) such that $L(p_0) = 1-\alpha$ & $L(p_1) = \beta$ in life testing procedure is to terminate the test at a specified time t. The probability of rejecting bad lot is P^* and the maximum number of defectives accepted is c .The acceptance single sampling plan for percentiles based on truncated life test is derived to get minimum sample size n for the given acceptance number c so that, $L(p_1) = \beta$ does not exceed 1-P*is applied in SkSP-3.A lot is said to be bad lot if the true A bad lot means that the true $100q^{th}$ percentile t_q is less than the predetermined percentile t_q^0. Therefore, the probability P^* is defined as the confidence limit of rejecting a bad lot i.e. accepting a good lot with $t_q < t_q^0$ is at least equal to P^*. The modus operandi of the proposed plan is as follows:

Step 1: fix $\theta = 2$ and generate the value of η for the specified percentile (10^{th}) from equation,

$$\eta = \sqrt{-2\ln(1-q^{\frac{1}{\theta}})} \tag{3}$$

Step 2: Randomly determine the value of i, f, specified $P^*$and the acceptance number c.

Step 3: choose the smallest sample size n from Table 1 of Pradeepa Veerakumari (2016).

The value of P can be calculated from SkSP-3 from the OC function of SSP for life test based on percentiles of ERD. Hence, the procedure Hence SkSP-2 with SSP for life test based on the percentiles of ERD as reference plan is usually specified by SSP for life tests based on the percentiles of ERD characterized by clearing interval i and sampling frequency f.

3.2 Operating characteristic function

OC function is the most applied techniques to measure the efficiency of the sampling plan and from where the probability of acceptance is derived. It gives the probability that the lot can be accepted. The OC function of SSP for life tests based on the percentiles of ERD is as follows,

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{4}$$

Where $p = F(t, \delta_0)$ represents the failure probability at time t given a determined $100q^{th}$ percentile of lifetime t_q^0 and p depends only on $\delta_0 = t/t_q^0$. The OC values are tabulated in Table 3 of Pradeepa Veerakumari (2016).

The OC function of SkSP-3 for the lot quality p are given by,

$$L(P) = \frac{[fP + (1-f)P^i(2-P^i)]}{[f + (1-f)P^i(2-P^i)]} \tag{5}$$

Then, the Average Sample number is

$$ASN(p) = ASN(R)F \tag{6}$$

Where, ASN(R) represents the Average Sample number of the reference plan, P represents the probability of acceptance of the reference plan. Fraction of lots inspection is given below:

$$F = \frac{f}{[f + (2-P^i)P^i(1-P)]} \tag{7}$$

Illustration 1:

Engineers experienced that the life time of the electric goods follows ERD. Skip lot sampling plan of type 2 with SSP as reference plan based on percentile is applied for testing. The parameters for the life testing is as follows: θ=2, $t = 40hrs$, $t_{0.1} = 20hrs$,c=2, α=0.05and β=0.10 then η=0.871929 from the equation and the ratio is found to be $t/t_{0.1} = 2.00$ By applying Table 1of Pradeepa Veerakumari (2016) the minimum sample size according to the requirements is $n = 7$ and the corresponding OC values $L(p)$ for the Single Sampling plan for the life tests based on percentiles of ERD $(n, c, t/t_{0.1}) = (7,2,1.5)$ with P^* = 0.90 under

ERD from Table 3 of Pradeepa Veerakumari (2016).are,

$t_{0.1}/t_{0.1}^{0}$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$L(p)$	0.0003	0.0601	0.3844	0.7281	0.9013	0.9662	0.9883	0.995	0.998

$L(p)$ is the P value for SkSP-3 with SSP for life tests based on the percentiles of ERD as reference plan. For $i=4$ and $f=1/3, k=4$; the probability of acceptance $L(p)$ values of SkSP-3 with SSP for life tests based on percentiles of ERD are found from eqn. 5 as,

$t_{0.1}/t_{0.1}^{0}$	1.00	1.25	1.50	1.75	2.00
$L(p)$	0.086	0.524	0.896	0.972	0.991

From the illustrations, it is indicated that the actual 10^{th} percentile is almost equal to the required 10th percentile $(t_{0.1}/t_{0.1}^{0} = 1.00)$ the producer's risk is approximately 0.9134 (1-0.0865). Also the producer's risk is nearly equal to 0.05 or less and the actual producer risk is large or nearly equal to 1.75 times of the required percentile. The OC curve is provided for the illustration as fig.1

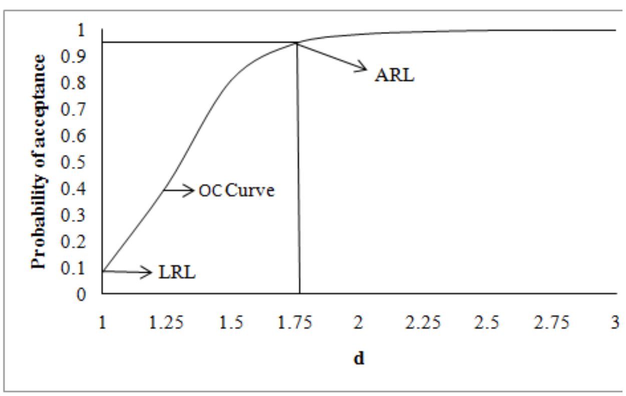

Figure :1.OC Curve for i=4, f=1/3,k=4, *P*=0.90*, *d=d*$_{0.1}$and θ=2

The figure 2 clearly says that the plan attains ARL when the actual life time percentile is in close proximity to 1.75 times greater than the specified 10^{th} percentile and attains LRL when the actual life time percentile is approximately equal to the specified life time percentile.

For the purpose of convenience OC values of the table are constructed and tabulated with parameters $i=4$, $f=1/3, k=4$ and $c=2$ in Table 1

4. Comparison of SkSP-3 with SSP for life tests based on percentiles of ERD as reference plan over SSP for life test based on percentiles of ERD

It is usual practice while comparing two plans comparing it by its OC curve and ASN values. The performance measures for instance ARL and LRL of the corresponding sampling plans are also used to compare the plans. Illustrations are provided for the comparison of Skip lot sampling plan with SSP for life tests based on percentiles of ERD as reference plan with SSP for life tests based on percentiles of ERD with the following illustration.

Illustration 2:

Consider that $\theta=2$, $t = 40hrs$, $t_{0.1} = 20hrs$, $c=2$, α = 0.05, β=0.10 then ,η=0.871929 is calculated from the equation 3.3 and the ratio, $t/t_{0.1} = 2.00$ and from Table 1 of Pradeepa Veerakumari (2016) the minimum sample size suitable for the given information is found to be as $n = 7$. And Table 3 of Pradeepa Veerakumari (2016) gives their respective OC values. With this plan as the reference plan the Skip lot sampling plan of type SkSP-3 is designed and their respective OC values are also tabulated in table 1 with $i=4$;k=4and $f=1/3$. The ASN is also found using the relation given in equation 6 . For the comparative purpose the values representing the illustration are tabulated in table 3.

The table 2 says that the OC values of SkSP -3 with SSP for life tests based on percentiles of ERD is slightly increased from SSP for life tests based on percentiles of ERD. The curves representing comparison of OC values for the defined parameters obtained from the table 2 are given in figure 3 respectively.

Table 2: OC values of SSP for life tests based on ERD percentiles and SkSP-3 with SSP for life tests based on ERD percentiles as reference plan

$t_{0.1} / t_{0.1}^{0}$	***OC Values***		**ASN *Values***	
	SSP ERD Percentiles	**SkSP-3 with SSP ERD Percentiles**	**SSP ERD Percentiles**	**SkSP-3 with SSP ERD Percentiles**
1	0.0864	0.0865	7	7
1.25	0.4485	0.5240	7	6
1.5	0.7742	0.8963	7	5
1.75	0.9223	0.9727	7	4
2	0.9743	0.9914	7	4
2.25	0.9913	0.9971	7	3
2.5	0.9969	0.9990	7	2
2.75	0.9988	0.9996	7	2
3	0.9995	0.9998	7	2

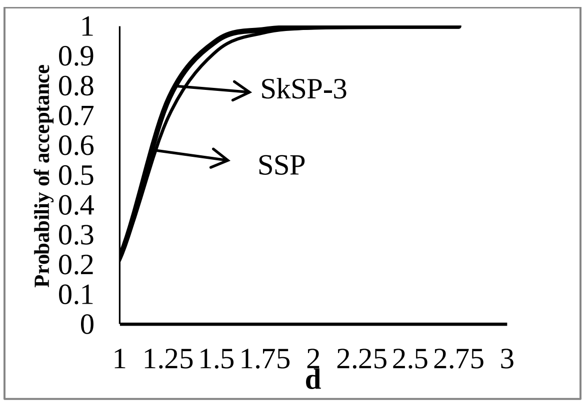

Figure 3. OC Curve of SkSP -3 with SSP for ERD percentiles and SSP for ERD percentiles

4. CONCLUSION

In this study Skip-lot Sampling plan of type-3 with SSP as reference plan based on the percentiles of ERD. SkSP-3 plan requires less ASN than the SSP results in reduction of inspection cost. The OC value of SkSP-3 with SSP as reference plan is little higher than the SSP.

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Table 1: OC values for SkSP- 3 with Single Sampling Plan (n, c=2, $t/t_{0.10}$) as reference plan for a given P^* under ERD when $\theta = 2$

P*	n	**t/t_q^0**	**t_q/t_q^0**								
			1.0000	**1.2500**	**1.5000**	**1.7500**	**2.0000**	**2.2500**	**2.5000**	**2.7500**	**3.0000**
0.7500	135.0000	0.7000	0.2603	0.8837	0.9813	0.9960	0.9990	0.9997	0.9999	1.0000	1.0000
0.7500	55.0000	0.9000	0.2595	0.8732	0.9789	0.9953	0.9988	0.9996	0.9999	1.0000	1.0000
0.7500	39.0000	1.0000	0.2474	0.8606	0.9765	0.9947	0.9986	0.9996	0.9999	1.0000	1.0000
0.7500	11.0000	1.5000	0.2504	0.8292	0.9680	0.9919	0.9978	0.9993	0.9998	0.9999	1.0000
0.7500	6.0000	2.0000	0.1674	0.6944	0.9380	0.9830	0.9947	0.9983	0.9994	0.9998	0.9999
0.7500	4.0000	2.5000	0.1485	0.6065	0.9072	0.9729	0.9908	0.9966	0.9987	0.9995	0.9998
0.9000	183.0000	0.7000	0.0995	0.7215	0.9594	0.9910	0.9977	0.9994	0.9998	0.9999	1.0000

0.9000	75.0000	0.9000	0.0959	0.6905	0.9530	0.9893	0.9972	0.9992	0.9998	0.9999	1.0000
0.9000	52.0000	1.0000	0.0969	0.6806	0.9502	0.9885	0.9970	0.9991	0.9997	0.9999	1.0000
0.9000	15.0000	1.5000	0.0830	0.5776	0.9226	0.9809	0.9945	0.9983	0.9994	0.9998	0.9999
0.9000	7.0000	2.0000	0.0866	0.5240	0.8963	0.9727	0.9914	0.9971	0.9990	0.9996	0.9998
0.9000	5.0000	2.5000	0.0419	0.3210	0.7749	0.9402	0.9799	0.9926	0.9971	0.9988	0.9995
0.9500	216.0000	0.7000	0.0498	0.5772	0.9373	0.9863	0.9965	0.9990	0.9997	0.9999	1.0000
0.9500	88.0000	0.9000	0.0486	0.5443	0.9278	0.9839	0.9957	0.9988	0.9996	0.9999	1.0000
0.9500	61.0000	1.0000	0.0491	0.5320	0.9233	0.9827	0.9953	0.9986	0.9996	0.9998	0.9999
0.9500	17.0000	1.5000	0.0464	0.4498	0.8877	0.9732	0.9922	0.9975	0.9992	0.9997	0.9999
0.9500	8.0000	2.0000	0.0436	0.3774	0.8374	0.9593	0.9871	0.9955	0.9984	0.9994	0.9998
0.9500	5.0000	2.5000	0.0419	0.3210	0.7749	0.9402	0.9799	0.9926	0.9971	0.9988	0.9995
0.9900	288.0000	0.7000	0.0100	0.3149	0.8600	0.9716	0.9924	0.9978	0.9993	0.9998	0.9999
0.9900	117.0000	0.9000	0.0097	0.2895	0.8376	0.9667	0.9909	0.9973	0.9991	0.9997	0.9999
0.9900	81.0000	1.0000	0.0098	0.2800	0.8266	0.9642	0.9901	0.9970	0.9990	0.9997	0.9998
0.9900	23.0000	1.5000	0.0073	0.20246	0.7208	0.9392	0.9823	0.9942	0.9979	0.9993	0.9997
0.9900	11.0000	2.0000	0.0048	0.1383	0.5725	0.8929	0.9680	0.9887	0.9958	0.9983	0.9993
0.9900	7.0000	2.5000	0.0027	0.0866	0.4084	0.8055	0.9415	0.9785	0.9914	0.9963	0.9985